CHAPTER 10

Study Guide and Review

Download Vocabulary Review from algebra2.com

FOLDABLES Study Organizer — GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.

Key Concepts

Midpoint and Distance Formulas (Lesson 10-1)

- $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Circles (Lesson 10-2)

- The equation of a circle with center (h, k) and radius r can be written in the form $(x - h)^2 + (y - k)^2 = r^2$.

Parabolas (Lesson 10-3)

Standard Form	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Axis of Symmetry	$x = h$	$y = k$

Ellipses (Lesson 10-4)

Standard Form of Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Direction of Major Axis	horizontal	vertical

Hyperbolas (Lesson 10-5)

Standard Form	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Transverse Axis	horizontal	vertical

Solving Quadratic Systems (Lesson 10-7)

- Systems of quadratic equations can be solved using substitution and elimination.
- A system of quadratic equations can have zero, one, two, three, or four solutions.

Key Vocabulary

asymptote (p. 591)
center of a circle (p. 574)
center of a hyperbola (p. 591)
center of an ellipse (p. 582)
circle (p. 574)
conic section (p. 567)
conjugate axis (p. 591)
directrix (p. 567)
ellipse (p. 581)
foci of a hyperbola (p. 590)
foci of an ellipse (p. 581)
focus of a parabola (p. 567)
hyperbola (p. 590)
latus rectum (p. 569)
major axis (p. 582)
minor axis (p. 582)
parabola (p. 567)
transverse axis (p. 591)
vertex of a hyperbola (p. 591)

Vocabulary Check

Tell whether each statement is *true* or *false*. If the statement is false, correct it to make it true.

1. An ellipse is the set of all points in a plane such that the sum of the distances from two given points in the plane, called the foci, is constant.
2. The major axis is the longer of the two axes of symmetry of an ellipse.
3. A parabola is the set of all points that are the same distance from a given point called the directrix and a given line called the focus.
4. The radius is the distance from the center of a circle to any point on the circle.
5. The conjugate axis of a hyperbola is a line segment parallel to the transverse axis.
6. A conic section is formed by slicing a double cone by a plane.
7. The set of all points in a plane that are equidistant from a given point in a plane, called the center, forms a circle.

Math Online Vocabulary Review at algebra2.com

CHAPTER 10

Study Guide and Review

Lesson-by-Lesson Review

10–1 Midpoint and Distance Formulas (pp. 562–566)

Find the midpoint of the line segment with endpoints at the given coordinates.

8. (1, 2), (4, 6)
9. (−8, 0), (−2, 3)
10. $\left(\frac{3}{5}, -\frac{7}{4}\right), \left(\frac{1}{4}, -\frac{2}{5}\right)$
11. (13, 24), (19, 28)

Find the distance between each pair of points with the given coordinates.

12. (−2, 10), (−2, 13)
13. (8, 5), (−9, 4)
14. (7, −3), (1, 2)
15. $\left(\frac{5}{4}, \frac{1}{2}\right), \left(\frac{1}{4}, \frac{3}{2}\right)$

HIKING For Exercises 16 and 17, use the following information.

Marc wants to hike from his camp to a waterfall. The waterfall is 5 miles south and 8 miles east of his campsite.

16. How far away is the waterfall?
17. Marc wants to stop for lunch halfway to the waterfall. If the camp is at the origin, where should he stop?

Example 1 **Find the midpoint of a segment whose endpoints are at (−5, 9) and (11, −1).**

Let $(x_1, y_1) = (-5, 9)$ and $(x_2, y_2) = (11, -1)$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5 + 11}{2}, \frac{9 + (-1)}{2}\right)$$

$$= \left(\frac{6}{2}, \frac{8}{2}\right) \text{ or } (3, 4) \quad \text{Simplify.}$$

Example 2 **Find the distance between $P(6, -4)$ and $Q(-3, 8)$. Let $(x_1, y_1) = (6, -4)$ and $(x_2, y_2) = (-3, 8)$.**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$= \sqrt{(-3 - 6)^2 + [8 - (-4)]^2}$$

$$= \sqrt{81 + 144} \quad \text{Subtract.}$$

$$= \sqrt{225} \text{ or } 15 \text{ units} \quad \text{Simplify.}$$

10–2 Parabolas (pp. 567–573)

(continued on the next page)

Identify the coordinates of the vertex and focus, the equation of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.

18. $(x - 1)^2 = 12(y - 1)$
19. $y + 6 = 16(x - 3)^2$
20. $x^2 - 8x + 8y + 32 = 0$
21. $x = 16y^2$

22. Write an equation for a parabola with vertex (0, 1) and focus (0, −1). Then graph the parabola.

Example 3 **Graph $4y - x^2 = 14x - 27$.**

Write the equation in the form $y = a(x - h)^2 + k$ by completing the square.

$$4y = x^2 + 14x - 27 \quad \text{Isolate the terms with } x.$$

$$4y = (x^2 + 14x + \blacksquare) - 27 - \blacksquare$$

$$4y = (x^2 + 14x + 49) - 27 - 49$$

$$4y = (x + 7)^2 - 76 \quad x^2 + 14x + 49 = (x + 7)^2$$

$$y = \frac{1}{4}(x + 7)^2 - 19 \quad \text{Divide each side by 4.}$$

10–2 Parabolas (pp. 567–573)

23. SPORTS When a golf ball is hit, the path it travels is shaped like a parabola. Suppose a golf ball is hit from ground level, reaches a maximum height of 100 feet, and lands 400 feet away. Assuming the ball was hit at the origin, write an equation of the parabola that models the flight of the ball.

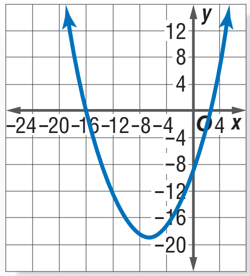
vertex: $(-7, -19)$

axis of symmetry: $x = -7$

direction of opening: upward since $a > 0$

focus: $\left(-7, -19 + \frac{1}{4\left(\frac{1}{4}\right)}\right)$ or $(-7, -18)$

directrix: $y = -19 - \frac{1}{4\left(\frac{1}{4}\right)}$ or $y = -20$



10–3 Circles (pp. 574–579)

Write an equation for the circle that satisfies each set of conditions.

24. center $(2, -3)$, radius 5 units

25. center $(-4, 0)$, radius $\frac{3}{4}$ units

26. endpoints of a diameter at $(9, 4)$ and $(-3, -2)$

27. center at $(-1, 2)$, tangent to x-axis

Find the center and radius of the circle with the given equation. Then graph the circle.

28. $x^2 + y^2 = 169$

29. $(x + 5)^2 + (y - 11)^2 = 49$

30. $x^2 + y^2 - 6x + 16y - 152 = 0$

31. $x^2 + y^2 + 6x - 2y - 15 = 0$

32. WEATHER On average the circular eye of a tornado is about 200 feet in diameter. Suppose a satellite photo showed the center of its eye at the point $(72, 39)$. Write an equation to represent the possible boundary of a tornado's eye.

Example 4 Graph $x^2 + y^2 + 8x - 24y + 16 = 0$.

First write the equation in the form $(x - h)^2 + (y - k)^2 = r^2$.

$$x^2 + y^2 + 8x - 24y + 16 = 0$$

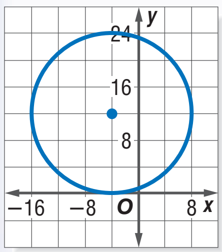
$$x^2 + 8x + \blacksquare + y^2 + -24y + \blacksquare = -16 + \blacksquare + \blacksquare$$

$$x^2 + 8x + 16 + y^2 + -24y + 144 = -16 + 16 + 144$$

$$(x + 4)^2 + (y - 12)^2 = 144$$

The center of the circle is at $(-4, 12)$ and the radius is 12.

Now draw the graph.



CHAPTER 10

Study Guide and Review

10–4 Ellipses (pp. 581–588)

33. Write an equation for the ellipse with endpoints of the major axis at (4, 1) and (−6, 1) and endpoints of the minor axis at (−1, 3) and (−1, −1).

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

34. $\frac{x^2}{16} + \frac{y^2}{25} = 1$

35. $\frac{(x+2)^2}{16} + \frac{(y-3)^2}{9} = 1$

36. $x^2 + 4y^2 - 2x + 16y + 13 = 0$

37. The Oval Office in the White House is an ellipse. The major axis is 10.9 meters and the minor axis is 8.8 meters. Write an equation to model the Oval Office. Assume that the origin is at the center of the Oval Office.

Example 5 **Graph $x^2 + 3y^2 - 16x + 24y + 31 = 0$.**

First write the equation in standard form by completing the squares.

$$x^2 + 3y^2 - 16x + 24y + 31 = 0$$

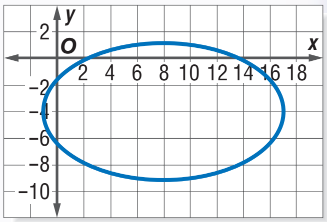
$$x^2 - 16x + {\color{red}\blacksquare} + 3(y^2 + 8y + {\color{cyan}\blacksquare}) = -31 + {\color{red}\blacksquare} + 3({\color{cyan}\blacksquare})$$

$$x^2 - 16x + {\color{red}64} + 3(y^2 + 8y + {\color{cyan}16}) = -31 + {\color{red}64} + 3({\color{cyan}16})$$

$$(x-8)^2 + 3(y+4)^2 = 81$$

$$\frac{(x-8)^2}{81} + \frac{(y+4)^2}{27} = 1$$

The center of the ellipse is at (8, −4). The length of the major axis is 18, and the length of the minor axis is $6\sqrt{3}$.



10–5 Hyperbolas (pp. 590–597)

(continued on the next page)

38. Write an equation for a hyperbola that has vertices at (2, 5) and (2, 1) and a conjugate axis of length 6 units.

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

39. $\frac{y^2}{4} - \frac{x^2}{9} = 1$

40. $16x^2 - 25y^2 - 64x - 336 = 0$

41. $\frac{(x-2)^2}{1} - \frac{(y+1)^2}{9} = 1$

42. $9y^2 - 16x^2 = 144$

Example 6 **Graph $9x^2 - 4y^2 + 18x + 32y - 91 = 0$.**

Complete the square for each variable to write this equation in standard form.

$$9x^2 - 4y^2 + 18x + 32y - 91 = 0$$

$$9(x^2 + 2x + {\color{red}\blacksquare}) - 4(y^2 - 8y + {\color{cyan}\blacksquare}) = 91 + 9({\color{red}\blacksquare}) - 4({\color{cyan}\blacksquare})$$

$$9(x^2 + 2x + {\color{red}1}) - 4(y^2 - 8y + {\color{cyan}16}) = 91 + 9({\color{red}1}) - 4({\color{cyan}16})$$

$$9(x+1)^2 - 4(y-4)^2 = 36$$

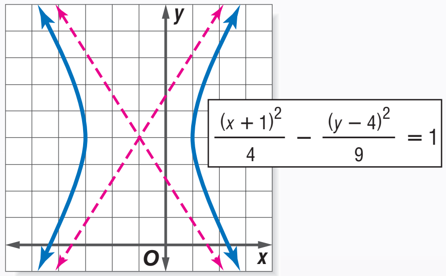
$$\frac{(x+1)^2}{4} - \frac{(y-4)^2}{9} = 1$$

Mixed Problem Solving
For mixed problem-solving practice, see page 935.

10–5 Hyperbolas (pp. 590–597)

43. MIRRORS A hyperbolic mirror is a mirror in the shape of one branch of a hyperbola. Such a mirror reflects light rays directed at one focus toward the other focus. Suppose a hyperbolic mirror is modeled by the upper branch of the hyperbola with equation $\frac{y^2}{9} - \frac{x^2}{16} = 1$. A light source is located at $(-10, 0)$. Where should the light from the source hit the mirror so that the light will be reflected to $(0, -5)$?

The center is at $(-1, 4)$. The vertices are at $(-3, 4)$ and $(1, 4)$ and the foci are at $(-1 \pm \sqrt{13}, 4)$. The equations of the asymptotes are $y - 4 = \pm\frac{3}{2}(x + 1)$.



10–6 Conic Sections (pp. 598–602)

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

44. $-4x^2 + y^2 + 8x - 8 = 0$

45. $x^2 + 4x - y = 0$

46. $x^2 + y^2 - 4x - 6y + 4 = 0$

47. $9x^2 + 4y^2 = 36$

Without writing the equation in standard form, state whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

48. $7x^2 + 9y^2 = 63$

49. $5y^2 + 2y + 4x - 13x^2 = 81$

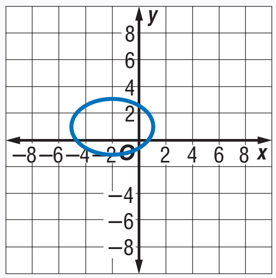
50. $x^2 - 8x + 16 = 6y$

51. $x^2 + 4x + y^2 - 285 = 0$

52. ASTRONOMY A satellite travels in a hyperbolic orbit. It reaches a vertex of its orbit at $(9, 0)$ and then travels along a path that gets closer and closer to the line $y = \frac{2}{9}x$. Write an equation that describes the path of the satellite if the center of its hyperbolic orbit is at $(0, 0)$.

Example 7 **Without writing the equation in standard form, state whether the graph of $4x^2 + 9y^2 + 16x - 18y - 11 = 0$ is a *parabola*, *circle*, *ellipse*, or *hyperbola*.**

In this equation, $A = 4$ and $C = 9$. Since A and C are both positive and $A \neq C$, the graph is an ellipse.



10–7 Solving Quadratic Systems (pp. 603–608)

Find the exact solution(s) of each system of equations.

53. $x^2 + y^2 - 18x + 24y + 200 = 0$
$4x + 3y = 0$

54. $4x^2 + y^2 = 16$
$x^2 + 2y^2 = 4$

Solve each system of inequalities by graphing.

55. $y < x$
$y > x^2 - 4$

56. $x^2 + y^2 \leq 9$
$x^2 + 4y^2 \leq 16$

57. ARCHITECTURE An architect is building the front entrance of a building in the shape of a parabola with the equation $y = -\frac{1}{10}(x - 10)^2 + 20$. While the entrance is being built the construction team puts in two support beams with equations $y = -x + 10$ and $y = x - 10$. Where do the support beams meet the parabola?

Example 8 **Solve the system of equations.**

$\mathbf{x^2 + y^2 + 2x - 12y + 12 = 0}$
$\mathbf{y + x = 0}$

Use substitution to solve the system.
First, rewrite $y + x = 0$ as $y = -x$.

$$x^2 + y^2 + 2x - 12y + 12 = 0$$
$$x^2 + (-x)^2 + 2x - 12(-x) + 12 = 0$$
$$2x^2 + 14x + 12 = 0$$
$$x^2 + 7x + 6 = 0$$
$$(x + 6)(x + 1) = 0$$

$x + 6 = 0$ or $x + 1 = 0$ Zero Product Property
$x = -6$ $\quad x = -1$ Solve for x.

Now solve for y.

$y = -x$ $\quad y = -x$ Equation for y in terms of x
$= -(-6)$ $\quad = -(-1)$ Substitute.
$= 6$ $\quad = 1$

The solutions of the system are $(-6, 6)$ and $(-1, 1)$.